7.2 Rational Exponents.

Objectives

1) Write as radical and evaluate

   $a^{1/n}$

   $a^{m/n}$

   $a^{-m/n}$

2) Review properties of exponents.

3) Use rational exponents and exponent laws to simplify radical expressions.

$$\boxed{a^{1/n} = \sqrt[n]{a}}$$

This means $3^{1/2} = \sqrt{3}$

$4^{1/3} = \sqrt[3]{4}$

$2^{1/4} = \sqrt[4]{2}$

Write each expression in radical notation, then evaluate and round to the nearest hundredth if appropriate

1. $9^{1/2} = \sqrt{9} = \boxed{3}$
2. $18^{1/5} = \sqrt[5]{18} \approx \boxed{1.78}$
3. $x^{1/4} = \boxed{\sqrt[4]{x}}$
4. $(3x)^{1/2} = \boxed{\sqrt{3x}}$
5. $3x^{1/2} = \boxed{3\sqrt{x}}$

NOTE: Denominator of a rational (fraction) exponent is ALWAYS the index of the radical.

$$\boxed{a^{m/n} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m}$$

This means $3^{2/5} = \sqrt[5]{3^2} = \sqrt[5]{9}$

Write each expression in radical notation and evaluate.

6. $(-8)^{2/3} = \sqrt[3]{(-8)^2} = \sqrt[3]{64} = \boxed{4}$

   OR $= (\sqrt[3]{-8})^2 = (-2)^2 = \boxed{4}$

7. $10^{3/4} = \sqrt[4]{10^3} = \boxed{\sqrt[4]{1000}} \approx \boxed{5.62}$

GC: $18^{1/5} = 18\text{^}(1/5)$

$18^{1/5} = 18^{.2} = 18\text{^}.2$

order of op: must divide before exponent, So extra parentheses needed

Recall exponent laws: $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$

(negative exponents move the base from numerator to denominator .... or from denominator to numerator)

This means ...

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(\sqrt[n]{a})^m} = \frac{1}{\sqrt[n]{a^m}}$$

and $$\frac{1}{a^{-m/n}} = a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Write each expression in radical notation and evaluate.

(8) $16^{-1/4}$

$= \frac{1}{16^{1/4}}$ move negative exponent to positive exp in denom

$= \frac{1}{\sqrt[4]{16}}$ write as radical

$= \boxed{\frac{1}{2}}$ simplify $2^4 = 16$ so $\sqrt[4]{16} = 2$

(9) $27^{-2/3}$

$= \frac{1}{27^{2/3}}$ move

$= \frac{1}{(\sqrt[3]{27})^2}$ radical

$= \frac{1}{3^2}$ evaluate inside

$= \boxed{\frac{1}{9}}$ then exponent

Exponent Laws

Exponent law #1 $a^n \cdot a^m = a^{n+m}$ Example: $x^2 \cdot x^3 = x^{2+3} = x^5$

With fraction exponents, it works the same

- Same base, appears twice
- Multiplied bases
- Add exponents

$$a^{p/q} \cdot a^{r/t} = a^{p/q + r/t}$$

Fraction example: $x^{2/3} \cdot x^{1/6} = x^{2/3 + 1/6} = x^{5/6}$

Caution: Be patient! You know how to add fractions, but it may take an extra step of work, or careful use of your calculator.

Exponent law #2 $\dfrac{a^n}{a^m} = a^{n-m}$ Examples:

$$\frac{x^3}{x^2} = x^{3-2} = x^1 = x$$

$$\frac{x^4}{x^7} = \frac{1}{x^{7-4}} = \frac{1}{x^3} = x^{-3}$$

With fraction exponents, it works the same

- Same base, appears twice
- Divided bases
- Subtract exponents, numerator exponent minus denominator $\rightarrow$ result in numerator
- OR Subtract exponents, denominator exponent minus numerator $\rightarrow$ result in denominator

$$\frac{a^{p/q}}{a^{r/t}} = a^{p/q - r/t}$$

Fraction examples:

$$\frac{x^{2/3}}{x^{1/6}} = x^{2/3 - 1/6} = x^{3/6} = x^{1/2}$$

$$\frac{x^{1/4}}{x^{3/4}} = \frac{1}{x^{3/4 - 1/4}} = \frac{1}{x^{2/4}} = \frac{1}{x^{1/2}}$$

Exponent law #3 $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$ Examples:

$$x^{-3} = \frac{1}{x^3}$$

$$\frac{1}{x^{-3}} = x^3$$

With fraction exponents, it works the same

- Negative exponent
- Move exponent from numerator to denominator (or denominator to numerator) of fraction, change sign of exponent to positive

$$a^{-p/q} = \frac{1}{a^{p/q}}$$

Fraction example: $x^{-2/3} = \dfrac{1}{x^{2/3}}$

$$\frac{1}{a^{-p/q}} = a^{p/q} \qquad \text{Fraction example:} \qquad \frac{1}{x^{-2/3}} = x^{2/3}$$

Exponent law #4 $a^0 = 1$ Examples: $1 = \frac{x^2}{x^2} = x^{2-2} = x^0 = 1$

Fraction example: $1 = \frac{x^{2/3}}{x^{2/3}} = x^{2/3 - 2/3} = x^0 = 1$

<u>Caution</u>: Zero exponent resolves to the number 1. There is no base anymore!

Exponent law #5 $(a^n)^m = a^{n \cdot m}$ Example: $(x^{-3})^2 = x^{-3 \cdot 2} = x^{-6} = \frac{1}{x^6}$

With fraction exponents, it works the same

- One base
- Parentheses separate two exponents
- Multiply exponents

$$\left(a^{p/q}\right)^{r/t} = a^{p/q \cdot r/t} \qquad \text{Fraction example:} \qquad \left(x^{2/3}\right)^{1/6} = x^{2/3 \cdot 1/6} = x^{1/9}$$

Exponent law #6 $(ab)^n = a^n b^n$ Examples: $(xy)^{-2} = x^{-2}y^{-2} = \frac{1}{x^2y^2}$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \qquad \left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3}$$

With fraction exponents, it works the same

- Two bases, inside parentheses
- One exponent outside parentheses
- "Share" exponents, each base inside parentheses gets the exponent outside those parentheses

$$(ab)^{p/q} = a^{p/q} b^{p/q} \qquad \text{Fraction examples:} \qquad (xy)^{2/3} = x^{2/3} y^{2/3}$$

$$\left(\frac{a}{b}\right)^{p/q} = \frac{a^{p/q}}{b^{p/q}} \qquad \left(\frac{x}{y}\right)^{2/3} = \frac{a^{2/3}}{b^{2/3}}$$

$$\left(\frac{a}{b}\right)^{-p/q} = \left(\frac{b}{a}\right)^{p/q} = \frac{b^{p/q}}{a^{p/q}} \qquad \left(\frac{x}{y}\right)^{-2/3} = \left(\frac{y}{x}\right)^{2/3} = \frac{y^{2/3}}{x^{2/3}}$$

<u>Caution</u>: No math term exists for what we do with these exponents. Do <u>not</u> use the word "distribute", which refers specifically to multiplying a sum, such as $3(2+5) = 3 \cdot 2 + 3 \cdot 5$

At this point in chapter 7, we will begin using variables in expressions where the signs could be complicated to analyze.

Though this is a useful skill, we won't expect you to do it in this level of class.

So... sometimes we'll want you to be aware of absolute values, and sometimes not.

This will appear in the instructions.

$\boxed{\text{"Assume that all variables can be any real number."}}$

↳ This means we might have negative numbers, and might need absolute values. So use absolute values correctly.

$\boxed{\text{"Assume that all variables are positive."}}$

↳ This means don't worry about absolute values.

Assume all variables are positive numbers.
Write each expression using rational exponents and simplify.
a) write final answer with positive exponents.
b) Then write final answer with radical notation.

(10) $\sqrt[3]{x} \cdot \sqrt[4]{x}$

$= x^{1/3} \cdot x^{1/4}$ write each radical with exponent

$= x^{1/3+1/4}$ same base, mult $\Rightarrow$ add exponent $x^n \cdot x^m = x^{n+m}$

$= \boxed{x^{7/12}}$ a) $\frac{1}{3}+\frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

$= \boxed{\sqrt[12]{x^7}}$ b)

(11) $\sqrt[3]{64x^2}$

$= (64x^2)^{1/3}$ write with exponent

$= 64^{1/3} \cdot (x^2)^{1/3}$ exponent law $(ab)^n = a^n b^n$

$= \sqrt[3]{64} \cdot x^{2 \cdot 1/3}$ simplify $\sqrt[3]{64} = 4$ exponent law $(x^n)^m = x^{n \cdot m}$

$= \boxed{4x^{2/3}}$ a)

$= \boxed{4\sqrt[3]{x^2}}$ b)

(12) $\dfrac{\sqrt[4]{81x}}{\sqrt{x}}$

$= \dfrac{(81x)^{1/4}}{x^{1/2}}$ write with exponent

$= \dfrac{(81)^{1/4} \cdot x^{1/4}}{x^{1/2}}$ exponent law $(ab)^n = a^n b^n$

$= \sqrt[4]{81} \cdot x^{1/4 - 1/2}$ simplify $\sqrt[4]{81} = 3$ exponent law $\frac{x^n}{x^m} = x^{n-m}$

$= 3x^{-1/4}$

cont...

$= 3 \cdot \frac{1}{x^{1/4}}$ write with positive exp

$= \boxed{\frac{3}{x^{1/4}}}$ a)

$= \boxed{\frac{3}{\sqrt[4]{x}}}$ b)

(13) $\left(\frac{x^2}{64}\right)^{-1/2}$

$= \left(\frac{64}{x^2}\right)^{1/2}$ exponent law $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$= \frac{64^{1/2}}{(x^2)^{1/2}}$ exponent law $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$= \frac{\sqrt{64}}{x^{2 \cdot \frac{1}{2}}}$ simplify $\sqrt{64}$

exponent law $(x^n)^m = x^{nm}$

$= \frac{8}{x^1}$

$= \boxed{\frac{8}{x}}$ a) and b)

(14) $\sqrt{\sqrt[7]{x-3}}$

$= \sqrt{(x-3)^{1/7}}$

$= \left((x-3)^{1/7}\right)^{1/2}$

$= (x-3)^{1/7 \cdot 1/2}$

$= \boxed{(x-3)^{1/14}}$ a) $= \boxed{\sqrt[14]{x-3}}$ b)

(15) $\sqrt[3]{c^9}$

$= (c^9)^{1/3}$ exponent

$= c^{9 \cdot \frac{1}{3}}$ exponent law $(a^n)^m = a^{nm}$

$= \boxed{c^3}$ a) and b) mult $9 \cdot \frac{1}{3} = 3$

(16) $\dfrac{y^{-1/4}}{x^{-1/2}}$

$= \dfrac{1}{y^{1/4}} \cdot x^{1/2}$ move neg exp to make positive

$= \boxed{\dfrac{x^{1/2}}{y^{1/4}}}$ a)

$= \boxed{\dfrac{\sqrt{x}}{\sqrt[4]{y}}}$ b)

(17) $\sqrt{x}\,(1 + \sqrt{x})$

$= \sqrt{x} + (\sqrt{x})^2$ dist

$= \boxed{\sqrt{x} + x}$ b)

$= \boxed{x^{1/2} + x}$ a)